CALCULATION OF THE PROCESS OF FILLING A GAS CONTAINER

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Formulas are presented for calculating the process of filling a vessel with a gas. These formulas were obtained by integration of the differential equations of thermodynamics of a variable mass, allowing for variation in the parameters of the incoming gas, and for heat transfer between the walls and the gas.

References [1] and [3], dealing with the calculation of gas-vessel filling processes, make the assumption that the parameters of the entering gas are constant. Change in the flow rate of the gas is taken into account only through the increase in back pressure of the vessel being filled. Actually, the filling of pneumatic systems is associated in many cases with the emptying of other vessels. Therefore, the parameters of the incoming gas are a function of time. The relevant quantities—pressure, temperature, density, and flow rate—are determined by formulas obtained in [2]. We consider the solution of the problem of a vessel being filled with a gas when the parameters of the entering gas are known functions of time.

We write the differential equation describing variation of the gas parameters in the vessel while it is being filled [7]:

$$\frac{dp_2}{d\tau} = \frac{k-1}{V_2}\left(\frac{dQ_2}{d\tau} + i_1 G_1\right), \qquad (1)$$

$$\frac{dp_2}{d\rho_2} = \frac{k-1}{G_1}\left(\frac{dQ_2}{d\tau} + i_1 G_1\right). \qquad (2)$$

The variation of flow rate* and temperature of the gas arriving in the vessel is determined by the formulas [2]

$$\bar{G} = \frac{G_1}{G_{01}} = \frac{a}{\sqrt{\vartheta_2^\delta}}\,\frac{\mathrm{sh}^\delta(\omega\tau+\varepsilon)}{\mathrm{th}(\omega\tau+\varepsilon)}, \qquad (3)$$

$$\bar{T}_1 = \frac{T_1}{T_{01}} = a^2\,\mathrm{cth}^2(\omega\tau+\varepsilon). \qquad (4)$$

The constants a, ϑ_2, δ, ω, and ε appearing in these expressions are given by the formulas of [8] in terms of the parameters of the supply system.

Dividing (1) by (2), and considering the familiar expression for enthalpy and Eqs. (3) and (4), we obtain the differential equation

$$\frac{d\rho_2}{d\tau} = \frac{a}{\sqrt{\vartheta_2^\delta}}\,\frac{G_{01}}{V_2}\,\mathrm{sh}^{\delta-1}(\omega\tau+\varepsilon)\,\mathrm{ch}(\omega\tau+\varepsilon). \qquad (5)$$

Integrating over the range 0 to τ and from ρ_{02} to ρ_2 [5], we obtain

$$\rho_2 = \rho_{02} + \frac{V_1}{V_2}\,\frac{\rho_{10}}{\sqrt{\vartheta_2^\delta}}\left[\sqrt{\vartheta_2^\delta} - \mathrm{sh}^\delta(\omega\tau+\varepsilon)\right]. \qquad (6)$$

Now we shall determine the rate of heat supplied to the vessel being filled $dQ_2/d\tau$ appearing in Eqs. (1) and (2). Taking the wall temperature of the vessel to be constant and equal to T_{02} (see [2]), in accordance with [6], we can write

$$\frac{dQ_2}{d\tau} = \alpha(T_{02} - T_2)F_2. \qquad (7)$$

Because of the great velocity of the incoming gas, strong mixing occurs in the vessel being filled. Therefore, we can consider that the heat-transfer coefficient α is given by the formula for turbulent flow [6]:

$$\alpha = ZK_T K_L w^{0.8} D^{-0.2}. \qquad (8)$$

The function Z depends on the temperature and is proportional to the 0.8-th power of the pressure. Using an approximation based on experimental data for Z, and taking into account the temperature dependence of the correction for nonisothermicity, we can formulate an analytical expression of the form (see [4])

$$ZK_T = \chi_2\left(\frac{p}{T}\right)^{0.8}, \qquad (9)$$

where the coefficient χ_2 depends on the properties of the working substance (for air $\chi_2 = 3.53\cdot10^{-2}$ $\mathrm{kg}^{0.2}\cdot\mathrm{sec}^{-0.6}\cdot\mathrm{deg}^{-0.2}\cdot\mathrm{m}^{-0.8}$). Assuming the approximation

$$w = \frac{4G_1}{\pi D^2 \rho_2},$$

and taking account of Eq. (9), we find the value of the heat-transfer coefficient α and, thus, the heat supply rate

$$\frac{dQ_2}{d\tau} = \vartheta(T_{02} - T_2)G_1^{0.8},$$

where

$$\vartheta = 1.213\ \chi_2 K_L R^{0.8} D^{-1.8} F_2.$$

Now substituting this expression into (1), and taking account of Eqs. (3), (4), and (6), we obtain the differential equation

$$\frac{dp_2}{d\tau} = \varphi_1\left(\frac{\mathrm{sh}^\delta t}{\mathrm{th}\, t}\right)^{0.8} + \varphi_2\,\frac{\mathrm{sh}^\delta t}{\mathrm{th}^3 t} + \varphi_3\left(\frac{\mathrm{sh}^\delta t}{\mathrm{th}\, t}\right)^{0.8}\frac{p_2}{(b_1 - \mathrm{sh}^\delta t)}. \qquad (10)$$

*We assume, for simplicity, that the pressure drop is critical in both vessels.

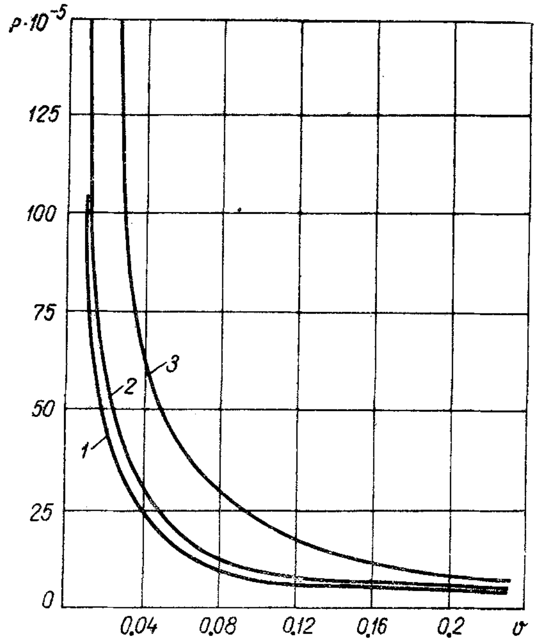

Fig. 1. Thermodynamic process of compression of a gas for various values of m: 1) isothermal process (m = 1): 2) compression of the gas during filling of the vessel with m = m(τ); 3) adiabatic process (m = 1.4).

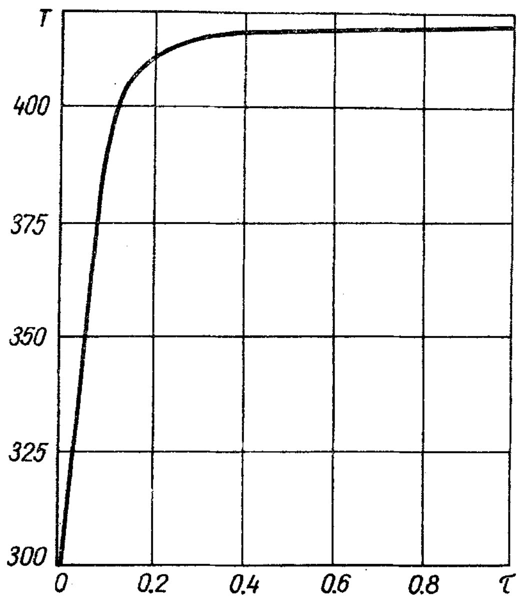

Fig. 2. Temperature of the gas (°K) in the vessel being filled as a function of time (sec).

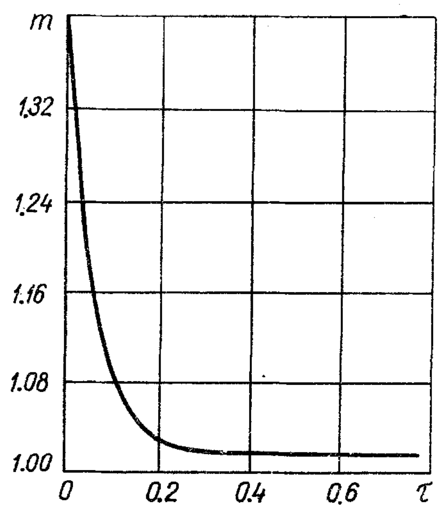

Fig. 3. Index for the thermodynamic process of compression of the gas in the vessel being filled as a function of time (sec).

Here

$$\varphi_1 = (k-1)\vartheta a^{0.8} T_{02} G_{01}^{0.8} \vartheta_2^{-0.46}/\omega V_2,$$

$$\varphi_2 = kA_k \mu F_{\text{кр}} \sqrt{RT_{01}\vartheta_2^{\delta}} p_{01} a^3/\omega V_2,$$

$$\varphi_3 = \frac{V_2}{V_1}\ \frac{\varphi_1}{\rho_{01} RT_{02}},\ b_1 = \frac{V_2}{V_1}\ \frac{\rho_{02}}{\rho_{01}} + \sqrt{\vartheta_2^{1-\delta}},$$

$$t = \omega\tau + \varepsilon.$$

Integration of this equation gives the result of [5],

$$p_2 = \exp(-\varphi_3 I_3)(\varphi_1 I_1 + \varphi_2 I_2 + C), \tag{11}$$

where

$$I_1 = \int \left(\frac{\operatorname{sh}^{\delta} t}{\operatorname{th} t}\right)^{0.8} \exp(\varphi_3 I_3)\, dt,$$

$$I_2 = \int \frac{\operatorname{sh}^{\delta} t}{\operatorname{th}^3 t} \exp(\varphi_3 I_3)\, dt,$$

$$I_3 = \int \left(\frac{\operatorname{sh}^{\delta} t}{\operatorname{th} t}\right)^{0.8} \frac{dt}{(b_1 - \operatorname{sh}^{\delta} t)},$$

and C is a constant determined by the initial conditions. The integrand of the last integral can be approximated by the following relation:

$$I_3' = \frac{dI_3}{d\tau} = B_1 + B_2 \exp(\lambda_1 t), \tag{12}$$

where B_1, B_2, and λ_1 are constants (for $\vartheta_2 = 3.38 \cdot 10^3$, $k = 1.4$, and $b_1 = 0.9941$; $B_1 = 53$; $B_2 = 20\,847$; $\lambda_1 = -0.61$). Equation (12) enables us to find the integral I_3:

$$I_3 = B_1 t + \frac{B_2}{\lambda_1} \exp(\lambda_1 t) + C_3. \tag{13}$$

The calculations presented indicate that the quantities $(\operatorname{sh}^{\delta} t \operatorname{cth} t)^{0.8}$ and $\operatorname{sh}^{\delta} t \operatorname{cth}^3 t$ for $\vartheta_2 \geq 200$ (this inequality holds in the majority of cases) are close to 1, i.e., practically $I_1 = I_2$. Evaluating the integral $I = I_1 = I_2$ numerically, we obtain a function which, in the range of possible variation of t, can be represented as follows:

for $\varepsilon \leqslant t \leqslant 6.65$

$$I = B_3 + B_4 t, \tag{14}$$

for $6.65 < t < \infty$

$$I = B_3' + B_4' \exp(\lambda_2 t). \tag{15}$$

Here B_3, B_3', B_4, B_4', λ_2 are constant coefficients (for $\varphi_3 = 2.31 \cdot 10^{-4}$; $B_3 = -5.053$; $B_3' = 1.116$; $B_4 = 16$; $B_4 = -25.9$; $\lambda_2 = -0.091$).

Thus, (13)–(15) allow us to find the pressure p_2 in the vessel as a function of the time duration of filling. Knowing the parameters of the working substance in the vessel being filled—the pressure p_2 (see (11)) and the density ρ_2 (see (6))—it is easy to determine the variation of the gas temperature T_2 from the equation of state.

When heat transfer is neglected, the solution of Eq. (10) is considerably simplified:

$$p_2 = p_{02} + \vartheta_4 + \frac{\varphi_2}{\delta} \operatorname{sh}^{\delta} t \left[1 + \frac{\operatorname{csch}^2 t}{k + (k-1)\vartheta_2}\right], \tag{16}$$

where

$$\vartheta_4 = \frac{k(k-1) G_{01} R \sqrt{T_{01}}}{\vartheta_1 (1 + \vartheta_2)[k + (k-1)\vartheta_2] V_2} \times$$
$$\times \{1 + \vartheta_2 [k + (k-1)\vartheta_2]\}.$$

The equation describing the thermodynamic process in the vessel being filled can be obtained from Eqs. (6) and (16):

$$p_2 + \beta_1 \rho_2 - \frac{\varphi_2}{(\delta - 2)} \left(b_1 - \frac{\beta_1 \delta}{\varphi_2} \rho_2\right)^{\gamma} - \beta_2 = 0. \tag{17}$$

Here

$$\beta_1 = \frac{\varphi_2 V_2 \vartheta_2^{\delta/2}}{V_1 \rho_{01} \delta},$$

$$\beta_2 = p_{02} + \vartheta_4 + \frac{\varphi_2 b_1}{\delta},$$

$$\gamma = k + (k-1)\vartheta_2.$$

Hence, it is a straightforward matter to determine an expression for the instantaneous index of the thermodynamic process (see [8]):

$$m = \frac{1}{2} [(k-1)\varphi_2 (1 + \vartheta_2)(b_1 - \operatorname{sh}^{\delta} t) \operatorname{cth}^2 t] \times \left[p_{02} + \vartheta_4 + \frac{\varphi_2}{\delta - 2} \operatorname{sh}^{\delta} t \times \left(\operatorname{cth}^2 t - \frac{2}{\delta}\right)\right]^{-1}, \tag{18}$$

or, approximately,

$$m \cong \beta_1 \frac{\rho_2}{p_2}.$$

From the formulas obtained, approximate graphs have been constructed (for $T_{01} = 300°$ K, $T_{02} = 300°$ K, $F_{crit} = 3.14 \cdot 10^{-6}$ m^2; $V_1 = 70.7 \cdot 10^{-3}$ m^3; $V_2 = 1 \cdot 10^{-3}$ m^3; $k = 1.4$; $\chi = \chi_1 \sqrt{T_1} = 5.7$ kg$^{1/3}$/sec$^{5/3}$ $\cdot$ m$^{2/3}$ $\cdot$ deg$^{5/6}$; $p_{01} = 1.96 \cdot 10^7$ N/m^2; $p_{02} = 9.8 \cdot 10^4$ N/m^2; $\rho_{01} = 222$ kg/m^3; $\rho_{02} = 1.11$ kg/m^3; $R = 295$ J/kg $\cdot$ deg).

Figure 1 shows a graph of the filling process on a pv-diagram. It can be seen that the thermodynamic process examined falls between isothermal and adiabatic. This confirms the viewpoint of Mamontov [7] that, for processes with varying mass the presence of the condition $dQ = 0$ does not imply that the process is adiabatic. Figure 2 shows the temperature in the vessel being filled as a function of time. It can be seen that the gas temperature tends to some constant value as time goes on. This is confirmed also by Fig. 3, a graph of the variation of the index m for the thermodynamic process as a function of time, which shows that m approaches 1.

NOTATION

p is the pressure; ρ is the density; T is the temperature; τ is time; k is the adiabatic exponent; m is the index of the thermodynamic process; V is the volume of the vessel; i is enthalpy; G is the mass flow rate of gas, per sec; Q is the heat supply; α is the heat-transfer coefficient; μ is the flow rate coefficient; F_{crit} is the area of the cross section determining the gas flow rate; F is the area of the inner surface of the vessel; R is the gas constant; χ is an experimental coefficient (see [6]); ω is the gas velocity; D is the diameter of the vessel being filled; K_L is the correction for flow stabilization; χ_2 is an experimental coefficient; L is the length of the vessel. The subscript 0 denotes parameters at the initial time; 1 denotes parameters in the vessel being emptied; 2 denotes parameters in the vessel being filled.

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